11-2: OPERATIONS WITH RADICAL EXPRESSIONS

Lesson Objectives:
- *Simplifying sums and differences of radicals*
- *Simplifying products and quotients of radicals*

For radical expressions, ***like radicals*** have the same radicand. ***Unlike radicals*** do not have the same radicand. For example, $4\sqrt{7}$ and $-12\sqrt{7}$ are like radicals, but $3\sqrt{11}$ and $2\sqrt{5}$ are unlike radicals. To simplify sums and differences, you use the Distributive Property to combine like radicals.

EXAMPLE 1: COMBINING LIKE RADICALS
Simplify.

1. $\sqrt{2}+3\sqrt{2}$
2. $-3\sqrt{5}-4\sqrt{5}$
3. $\sqrt{10}-5\sqrt{10}$
4. $16\sqrt{3}+3\sqrt{3}$
5. $-3\sqrt{6}+8\sqrt{6}$
6. $\sqrt{5}-3\sqrt{5}$
7. $6\sqrt{7}+4\sqrt{7}$
8. $-5\sqrt{3}-3\sqrt{3}$

EXAMPLE 2: SIMPLIFYING TO COMBINE LIKE RADICALS
Simplify.

9. $7\sqrt{3}-2\sqrt{12}$
10. $3\sqrt{20}+2\sqrt{5}$
11. $3\sqrt{3}-2\sqrt{27}$
12. $-3\sqrt{5}-2\sqrt{45}$
13. $\sqrt{18}+\sqrt{2}$
14. $\sqrt{8}-2\sqrt{2}$
15. $3\sqrt{7}-\sqrt{28}$
16. $-4\sqrt{10}+6\sqrt{40}$

EXAMPLE 3: USING THE DISTRIBUTIVE PROPERTY
Simplify.

17. $\sqrt{3}\left(\sqrt{6}+7\right)$
18. $\sqrt{5}\left(2+\sqrt{10}\right)$
19. $\sqrt{2x}\left(\sqrt{6x}-11\right)$
20. $\sqrt{5a}\left(\sqrt{5a}+3\right)$

21. $\sqrt{2}\left(\sqrt{8}-4\right)$
22. $2\sqrt{3}\left(\sqrt{3}-1\right)$
23. $\sqrt{3}\left(\sqrt{15}+2\right)$
24. $\sqrt{6}\left(\sqrt{6}-\sqrt{2}\right)$

EXAMPLE 4: SIMPLIFYING USING F.O.I.L.
Simplify.

25. $\left(\sqrt{5}-2\sqrt{15}\right)\left(\sqrt{5}+\sqrt{15}\right)$
26. $\left(2\sqrt{6}+3\sqrt{3}\right)\left(\sqrt{6}-5\sqrt{3}\right)$
27. $\left(2\sqrt{11}+5\right)\left(\sqrt{11}+2\right)$
28. $\left(\sqrt{7}+4\right)^2$

29. $\left(3\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}-5\sqrt{3}\right)$
30. $\left(2\sqrt{5}-\sqrt{6}\right)\left(4\sqrt{5}-3\sqrt{6}\right)$
31. $\left(2\sqrt{10}+\sqrt{3}\right)^2$
32. $\left(4-\sqrt{13}\right)\left(9+\sqrt{13}\right)$

Conjugates are the sum and the difference of the same two terms. The radical expressions $\sqrt{5}+\sqrt{2}$ and $\sqrt{5}-\sqrt{2}$ are conjugates. The product of two conjugates results in a difference of two squares.

$\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)=$

Notice that the product of these conjugates has no radical. You recall that a simplified radical expression has no radical in the denominator. When a denominator contains a sum or a difference including radical expressions, you can rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator. For example, to simplify a radical expression like $\dfrac{6}{\sqrt{5}-\sqrt{2}}$, you multiply by $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$.

EXAMPLE 5: RATIONALIZE A DENOMINATOR USING CONJUGATES
Simplify.

33. $\dfrac{6}{\sqrt{5}-\sqrt{2}}$

34. $\dfrac{4}{\sqrt{7}+\sqrt{5}}$

35. $\dfrac{-4}{\sqrt{20}+\sqrt{8}}$

36. $\dfrac{-5}{\sqrt{11}-\sqrt{3}}$

37. $\dfrac{8}{\sqrt{7}-\sqrt{3}}$

38. $\dfrac{3}{\sqrt{10}-\sqrt{5}}$

39. $\dfrac{48}{\sqrt{6}+\sqrt{18}}$

40. $\dfrac{9}{\sqrt{12}-\sqrt{11}}$