Exercises

In Problems 7–22, use properties of logarithms to find the exact value of each expression. Do not use a calculator.

7. $\log_3 3^{71}$
8. $\log_2 2^{-13}$
9. $\ln e^{-4}$
10. $\ln e^{\sqrt{2}}$
11. $2^{\log_2 7}$
12. $e^{\ln 8}$
13. $\log_8 2 + \log_8 4$
14. $\log_6 9 + \log_6 4$
15. $\log_6 18 - \log_6 3$
16. $\log_8 16 - \log_8 2$
17. $\log_2 6 \cdot \log_6 4$
18. $\log_3 8 \cdot \log_8 9$
19. $3^{\log_3 5 - \log_3 4}$
20. $5^{\log_5 6 + \log_5 7}$
21. $e^{\log_{e^2} 16}$
22. $e^{\log_{e^2} 9}$

In Problems 23–30, suppose that $\ln 2 = a$ *and* $\ln 3 = b$. *Use properties of logarithms to write each logarithm in terms of a and b.*

23. $\ln 6$
24. $\ln \frac{2}{3}$
25. $\ln 1.5$
26. $\ln 0.5$
27. $\ln 8$
28. $\ln 27$
29. $\ln \sqrt[5]{6}$
30. $\ln \sqrt[4]{\frac{2}{3}}$

In Problems 31–50, write each expression as a sum and/or difference of logarithms. Express powers as factors.

31. $\log_5(25x)$
32. $\log_3 \frac{x}{9}$
33. $\log_2 z^3$
34. $\log_7(x^5)$
35. $\ln(ex)$
36. $\ln \frac{e}{x}$
37. $\ln(xe^x)$
38. $\ln \frac{x}{e^x}$
39. $\log_a(u^2v^3), \quad u > 0, v > 0$
40. $\log_2\left(\frac{a}{b^2}\right), \quad a > 0, b > 0$
41. $\ln\left(x^2\sqrt{1-x}\right), \quad 0 < x < 1$
42. $\ln\left(x\sqrt{1+x^2}\right), \quad x > 0$
43. $\log_2\left(\frac{x^3}{x-3}\right), \quad x > 3$
44. $\log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right), \quad x > 1$
45. $\log\left[\frac{x(x+2)}{(x+3)^2}\right], \quad x > 0$
46. $\log\left[\frac{x^3\sqrt{x+1}}{(x-2)^2}\right], \quad x > 2$
47. $\ln\left[\frac{x^2-x-2}{(x+4)^2}\right]^{1/3}, \quad x > 2$
48. $\ln\left[\frac{(x-4)^2}{x^2-1}\right]^{2/3}, \quad x > 4$
49. $\ln\frac{5x\sqrt{1+3x}}{(x-4)^3}, x > 4$
50. $\ln\left[\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right], \quad 0 < x < 1$

In Problems 51–64, write each expression as a single logarithm.

51. $3\log_5 u + 4\log_5 v$
52. $2\log_3 u - \log_3 v$
53. $\log_3 \sqrt{x} - \log_3 x^3$
54. $\log_2\left(\frac{1}{x}\right) + \log_2\left(\frac{1}{x^2}\right)$
55. $\log_4(x^2-1) - 5\log_4(x+1)$
56. $\log(x^2+3x+2) - 2\log(x+1)$
57. $\ln\left(\frac{x}{x-1}\right) + \ln\left(\frac{x+1}{x}\right) - \ln(x^2-1)$
58. $\log\left(\frac{x^2+2x-3}{x^2-4}\right) - \log\left(\frac{x^2+7x+6}{x+2}\right)$
59. $8\log_2\sqrt{3x-2} - \log_2\left(\frac{4}{x}\right) + \log_2 4$
60. $21\log_3\sqrt[3]{x} + \log_3(9x^2) - \log_3 9$
61. $2\log_a(5x^3) - \frac{1}{2}\log_a(2x+3)$
62. $\frac{1}{3}\log(x^3+1) + \frac{1}{2}\log(x^2+1)$
63. $2\log_2(x+1) - \log_2(x+3) - \log_2(x-1)$
64. $3\log_5(3x+1) - 2\log_5(2x-1) - \log_5 x$

In Problems 65–72, use the Change-of-Base Formula and a calculator to evaluate each logarithm. Round your answer to three decimal places.

65. $\log_3 21$
66. $\log_5 18$
67. $\log_{1/3} 71$
68. $\log_{1/2} 15$
69. $\log_{\sqrt{2}} 7$
70. $\log_{\sqrt{5}} 8$
71. $\log_\pi e$
72. $\log_\pi \sqrt{2}$

In Problems 73–78, graph each function using a graphing utility and the Change-of-Base Formula.

73. $y = \log_4 x$
74. $y = \log_5 x$
75. $y = \log_2(x+2)$
76. $y = \log_4(x-3)$
77. $y = \log_{x-1}(x+1)$
78. $y = \log_{x+2}(x-2)$

In Problems 79–88, express y as a function of x. The constant C is a positive number.

79. $\ln y = \ln x + \ln C$

80. $\ln y = \ln(x + C)$

81. $\ln y = \ln x + \ln(x + 1) + \ln C$

82. $\ln y = 2\ln x - \ln(x + 1) + \ln C$

83. $\ln y = 3x + \ln C$

84. $\ln y = -2x + \ln C$

85. $\ln(y - 3) = -4x + \ln C$

86. $\ln(y + 4) = 5x + \ln C$

87. $3\ln y = \frac{1}{2}\ln(2x + 1) - \frac{1}{3}\ln(x + 4) + \ln C$

88. $2\ln y = -\frac{1}{2}\ln x + \frac{1}{3}\ln(x^2 + 1) + \ln C$

89. Find the value of $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8$.

90. Find the value of $\log_2 4 \cdot \log_4 6 \cdot \log_6 8$.

91. Find the value of $\log_2 3 \cdot \log_3 4 \cdot \cdots \cdot \log_n(n + 1) \cdot \log_{n+1} 2$.

92. Find the value of $\log_2 2 \cdot \log_2 4 \cdot \cdots \cdot \log_2 2^n$.

93. Show that $\log_a\left(x + \sqrt{x^2 - 1}\right) + \log_a\left(x - \sqrt{x^2 - 1}\right) = 0$.

94. Show that $\log_a\left(\sqrt{x} + \sqrt{x - 1}\right) + \log_a\left(\sqrt{x} - \sqrt{x - 1}\right) = 0$.

95. Show that $\ln(1 + e^{2x}) = 2x + \ln(1 + e^{-2x})$.

96. Difference Quotient If $f(x) = \log_a x$, show that $\dfrac{f(x + h) - f(x)}{h} = \log_a\left(1 + \dfrac{h}{x}\right)^{1/h}, \quad h \neq 0$.

97. If $f(x) = \log_a x$, show that $-f(x) = \log_{1/a} x$.

98. If $f(x) = \log_a x$, show that $f(AB) = f(A) + f(B)$.

99. If $f(x) = \log_a x$, show that $f\left(\dfrac{1}{x}\right) = -f(x)$.

100. If $f(x) = \log_a x$, show that $f(x^\alpha) = \alpha f(x)$.

101. Show that $\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$, where a, M, and N are positive real numbers, with $a \neq 1$.

102. Show that $\log_a\left(\dfrac{1}{N}\right) = -\log_a N$, where a and N are positive real numbers, with $a \neq 1$.

103. Graph $Y_1 = \log(x^2)$ and $Y_2 = 2\log(x)$ using a graphing utility. Are they equivalent? What might account for any differences in the two functions?