18. $-3\overline{)\begin{array}{rrrr}-4 & 5 & 0 & 8\\ & 12 & -51 & 153\\ \hline -4 & 17 & -51 & 161\end{array}}$

Remainder = $161 \neq 0$. Therefore, $x+3$ is not a factor of $-4x^3+5x^2+8$.

19. $2\overline{)\begin{array}{rrrrr}3 & -6 & 0 & -5 & 10\\ & 6 & 0 & 0 & -10\\ \hline 3 & 0 & 0 & -5 & 0\end{array}}$

Remainder = 0. Therefore, $x-2$ is a factor of $3x^4-6x^3-5x+10$.

20. $2\overline{)\begin{array}{rrrrr}4 & 0 & -15 & 0 & -4\\ & 8 & 16 & 2 & 4\\ \hline 4 & 8 & 1 & 2 & 0\end{array}}$

Remainder = 0. Therefore, $x-2$ is a factor of $4x^4-15x^2-4$.

21. $-3\overline{)\begin{array}{rrrrrrr}3 & 0 & 0 & 82 & 0 & 0 & 27\\ & -9 & 27 & -81 & -3 & 9 & -27\\ \hline 3 & -9 & 27 & 1 & -3 & 9 & 0\end{array}}$

Remainder = 0. Therefore, $x+3$ is a factor of $3x^6+82x^3+27$.

22. $-3\overline{)\begin{array}{rrrrrrr}2 & 0 & -18 & 0 & 1 & 0 & -9\\ & -6 & 18 & 0 & 0 & -3 & 9\\ \hline 2 & -6 & 0 & 0 & 1 & -3 & 0\end{array}}$

Remainder = 0. Therefore, $x+3$ is a factor of $2x^6-18x^4+x^2-9$.

23. $-4\overline{)\begin{array}{rrrrrrr}4 & 0 & -64 & 0 & 1 & 0 & -15\\ & -16 & 64 & 0 & 0 & -4 & 16\\ \hline 4 & -16 & 0 & 0 & 1 & -4 & 1\end{array}}$

Remainder = $1 \neq 0$. Therefore, $x+3$ is not a factor of $4x^6-64x^4+x^2-15$.

24. $-4\overline{)\begin{array}{rrrrrrrr}1 & 0 & -16 & 0 & 1 & 0 & -16\\ & -4 & 16 & 0 & 0 & -4 & 16\\ \hline 1 & -4 & 0 & 0 & 1 & -4 & 0\end{array}}$

Remainder = 0. Therefore, $x+4$ is a factor $x^6-16x^4+x^2-16$.

25. $\frac{1}{2}\overline{)\begin{array}{rrrrr}2 & -1 & 0 & 2 & -1\\ & 1 & 0 & 0 & 1\\ \hline 2 & 0 & 0 & 2 & 0\end{array}}$

Remainder = 0; therefore $x-\frac{1}{2}$ is a factor of $2x^4-x^3+2x-1$.

26. $-\frac{1}{3}\overline{)\begin{array}{rrrrr}3 & 1 & 0 & -3 & 1\\ & -1 & 0 & 0 & 1\\ \hline 3 & 0 & 0 & -3 & 2\end{array}}$

Remainder = $2 \neq 0$; therefore $x+\frac{1}{3}$ is not a factor of $3x^4+x^3-3x+1$.

27. $-2\overline{)\begin{array}{rrrr}1 & -2 & 3 & 5\\ & -2 & 8 & -22\\ \hline 1 & -4 & 11 & -17\end{array}}$

$\frac{x^3-2x^2+3x+5}{x+2} = x^2-4x+11+\frac{-17}{x+2}$

$a+b+c+d = 1-4+11-17 = -9$

28. Answers will vary.

Section R.7

1. lowest terms

2. Least Common Multiple

3. True; $\frac{2x^3-4x}{x-2} = \frac{2x\left(x^2-2\right)}{x-2}$

4. False;

$2x^3+6x^2=2x^2(x+3)$

$6x^4+4x^3=2x^3(3x+2)$

$LCM=2x^3(x+3)(3x+2)$

5. $\dfrac{3x+9}{x^2-9}=\dfrac{3(x+3)}{(x-3)(x+3)}=\dfrac{3}{x-3}$

6. $\dfrac{4x^2+8x}{12x+24}=\dfrac{4x(x+2)}{12(x+2)}=\dfrac{x}{3}$

7. $\dfrac{x^2-2x}{3x-6}=\dfrac{x(x-2)}{3(x-2)}=\dfrac{x}{3}$

8. $\dfrac{15x^2+24x}{3x^2}=\dfrac{3x(5x+8)}{3x^2}=\dfrac{5x+8}{x}$

9. $\dfrac{24x^2}{12x^2-6x}=\dfrac{24x^2}{6x(2x-1)}=\dfrac{4x}{2x-1}$

10. $\dfrac{x^2+4x+4}{x^2-4}=\dfrac{(x+2)(x+2)}{(x-2)(x+2)}=\dfrac{x+2}{x-2}$

11. $\dfrac{y^2-25}{2y^2-8y-10}=\dfrac{(y+5)(y-5)}{2(y^2-4y-5)}$

$=\dfrac{(y+5)(y-5)}{2(y-5)(y+1)}$

$=\dfrac{y+5}{2(y+1)}$

12. $\dfrac{3y^2-y-2}{3y^2+5y+2}=\dfrac{(3y+2)(y-1)}{(3y+2)(y+1)}=\dfrac{y-1}{y+1}$

13. $\dfrac{x^2+4x-5}{x^2-2x+1}=\dfrac{(x+5)(x-1)}{(x-1)(x-1)}=\dfrac{x+5}{x-1}$

14. $\dfrac{x-x^2}{x^2+x-2}=\dfrac{-x(x-1)}{(x+2)(x-1)}=\dfrac{-x}{x+2}=-\dfrac{x}{x+2}$

15. $\dfrac{x^2+5x-14}{2-x}=\dfrac{(x+7)(x-2)}{2-x}$

$=\dfrac{(x+7)(x-2)}{-1(x-2)}$

$=-(x+7)$

$=-x-7$

16. $\dfrac{2x^2+5x-3}{1-2x}=\dfrac{(2x-1)(x+3)}{-1(2x-1)}=-(x+3)=-x-3$

17. $\dfrac{3x+6}{5x^2}\cdot\dfrac{x}{x^2-4}=\dfrac{3(x+2)}{5x^2}\cdot\dfrac{x}{(x-2)(x+2)}$

$=\dfrac{3}{5x(x-2)}$

18. $\dfrac{3}{2x}\cdot\dfrac{x^2}{6x+10}=\dfrac{3}{2}\cdot\dfrac{x}{2(3x+5)}=\dfrac{3x}{4(3x+5)}$

19. $\dfrac{4x^2}{x^2-16}\cdot\dfrac{x^3-64}{2x}$

$=\dfrac{4x^2}{(x-4)(x+4)}\cdot\dfrac{(x-4)(x^2+4x+16)}{2x}$

$=\dfrac{2x\cdot 2x(x-4)(x^2+4x+16)}{2x(x-4)(x+4)}$

$=\dfrac{2x(x^2+4x+16)}{x+4}$

20. $\dfrac{12}{x^2+x}\cdot\dfrac{x^3+1}{4x-2}=\dfrac{12}{x(x+1)}\cdot\dfrac{(x+1)(x^2-x+1)}{2(2x-1)}$

$=\dfrac{2\cdot 6(x+1)(x^2-x+1)}{2x(x+1)(2x-1)}$

$=\dfrac{6(x^2-x+1)}{x(2x-1)}$

21. $\dfrac{4x-8}{-3x}\cdot\dfrac{12}{12-6x}=\dfrac{4(x-2)}{-3x}\cdot\dfrac{12}{6(2-x)}$

$=\dfrac{4(x-2)}{-3x}\cdot\dfrac{2}{(-1)(x-2)}$

$=\dfrac{8}{3x}$

22. $\dfrac{6x-27}{5x}\cdot\dfrac{2}{4x-18}=\dfrac{3(2x-9)}{5x}\cdot\dfrac{2}{2(2x-9)}=\dfrac{3}{5x}$

23. $\dfrac{x^2-3x-10}{x^2+2x-35}\cdot\dfrac{x^2+4x-21}{x^2+9x+14}$

$=\dfrac{(x-5)(x+2)}{(x+7)(x-5)}\cdot\dfrac{(x+7)(x-3)}{(x+7)(x+2)}$

$=\dfrac{x-3}{x+7}$

24. $\dfrac{x^2+x-6}{x^2+4x-5}\cdot\dfrac{x^2-25}{x^2+2x-15}$

$=\dfrac{(x-2)(x+3)}{(x+5)(x-1)}\cdot\dfrac{(x+5)(x-5)}{(x+5)(x-3)}$

$=\dfrac{(x-2)(x+3)(x-5)}{(x+5)(x-1)(x-3)}$

25. $\dfrac{\dfrac{6x}{x^2-4}}{\dfrac{3x-9}{2x+4}}=\dfrac{6x}{x^2-4}\cdot\dfrac{2x+4}{3x-9}$

$=\dfrac{6x}{(x-2)(x+2)}\cdot\dfrac{2(x+2)}{3(x-3)}$

$=\dfrac{4x}{(x-2)(x-3)}$

26. $\dfrac{\dfrac{12x}{5x+20}}{\dfrac{4x^2}{x^2-16}}=\dfrac{12x}{5x+20}\cdot\dfrac{x^2-16}{4x^2}$

$=\dfrac{12x}{5(x+4)}\cdot\dfrac{(x+4)(x-4)}{4x^2}$

$=\dfrac{3(x-4)}{5x}$

27. $\dfrac{\dfrac{8x}{x^2-1}}{\dfrac{10x}{x+1}}=\dfrac{8x}{x^2-1}\cdot\dfrac{x+1}{10x}$

$=\dfrac{8x}{(x-1)(x+1)}\cdot\dfrac{x+1}{10x}$

$=\dfrac{4}{5(x-1)}$

28. $\dfrac{\dfrac{x-2}{4x}}{\dfrac{x^2-4x+4}{12x}}=\dfrac{x-2}{4x}\cdot\dfrac{12x}{x^2-4x+4}$

$=\dfrac{x-2}{4x}\cdot\dfrac{12x}{(x-2)(x-2)}$

$=\dfrac{3}{x-2}$

29. $\dfrac{\dfrac{4-x}{4+x}}{\dfrac{4x}{x^2-16}}=\dfrac{4-x}{4+x}\cdot\dfrac{x^2-16}{4x}$

$=\dfrac{4-x}{4+x}\cdot\dfrac{(x+4)(x-4)}{4x}$

$=\dfrac{(4-x)(x-4)}{4x}$

$=-\dfrac{(x-4)^2}{4x}$

30. $\dfrac{\dfrac{3+x}{3-x}}{\dfrac{x^2-9}{9x^3}}=\dfrac{3+x}{3-x}\cdot\dfrac{9x^3}{x^2-9}$

$=\dfrac{3+x}{3-x}\cdot\dfrac{9x^3}{(x+3)(x-3)}$

$=\dfrac{9x^3}{(3-x)(x-3)}$

$=\dfrac{9x^3}{-(x-3)^2}$

$=-\dfrac{9x^3}{(x-3)^2}$

31. $\dfrac{\dfrac{x^2+7x+12}{x^2-7x+12}}{\dfrac{x^2+x-12}{x^2-x-12}}=\dfrac{x^2+7x+12}{x^2-7x+12}\cdot\dfrac{x^2-x-12}{x^2+x-12}$

$=\dfrac{(x+3)(x+4)}{(x-3)(x-4)}\cdot\dfrac{(x-4)(x+3)}{(x+4)(x-3)}$

$=\dfrac{(x+3)^2}{(x-3)^2}$

32. $\dfrac{\dfrac{x^2+7x+6}{x^2+x-6}}{\dfrac{x^2+5x-6}{x^2+5x+6}}=\dfrac{x^2+7x+6}{x^2+x-6}\cdot\dfrac{x^2+5x+6}{x^2+5x-6}$

$=\dfrac{(x+6)(x+1)}{(x+3)(x-2)}\cdot\dfrac{(x+2)(x+3)}{(x+6)(x-1)}$

$=\dfrac{(x+1)(x+2)}{(x-2)(x-1)}$

33. $$\frac{\dfrac{2x^2-x-28}{3x^2-x-2}}{\dfrac{4x^2+16x+7}{3x^2+11x+6}} = \frac{2x^2-x-28}{3x^2-x-2}\cdot\frac{3x^2+11x+6}{4x^2+16x+7}$$
$$= \frac{(2x+7)(x-4)}{(3x+2)(x-1)}\cdot\frac{(3x+2)(x+3)}{(2x+7)(2x+1)}$$
$$= \frac{(x-4)(x+3)}{(x-1)(2x+1)}$$

34. $$\frac{\dfrac{9x^2+3x-2}{12x^2+5x-2}}{\dfrac{9x^2-6x+1}{8x^2-10x-3}} = \frac{9x^2+3x-2}{12x^2+5x-2}\cdot\frac{8x^2-10x-3}{9x^2-6x+1}$$
$$= \frac{(3x+2)(3x-1)}{(3x+2)(4x-1)}\cdot\frac{(4x+1)(2x-3)}{(3x-1)(3x-1)}$$
$$= \frac{(4x+1)(2x-3)}{(4x-1)(3x-1)}$$

35. $$\frac{x}{2}+\frac{5}{2} = \frac{x+5}{2}$$

36. $$\frac{3}{x}-\frac{6}{x} = \frac{3-6}{x} = \frac{-3}{x} = -\frac{3}{x}$$

37. $$\frac{x^2}{2x-3}-\frac{4}{2x-3} = \frac{x^2-4}{2x-3} = \frac{(x+2)(x-2)}{2x-3}$$

38. $$\frac{3x^2}{2x-1}-\frac{9}{2x-1} = \frac{3x^2-9}{2x-1} = \frac{3\left(x^2-3\right)}{2x-1}$$

39. $$\frac{x+1}{x-3}+\frac{2x-3}{x-3} = \frac{x+1+2x-3}{x-3} = \frac{3x-2}{x-3}$$

40. $$\frac{2x-5}{3x+2}+\frac{x+4}{3x+2} = \frac{2x-5+x+4}{3x+2} = \frac{3x-1}{3x+2}$$

41. $$\frac{3x+5}{2x-1}-\frac{2x-4}{2x-1} = \frac{(3x+5)-(2x-4)}{2x-1}$$
$$= \frac{3x+5-2x+4}{2x-1}$$
$$= \frac{x+9}{2x-1}$$

42. $$\frac{5x-4}{3x+4}-\frac{x+1}{3x+4} = \frac{(5x-4)-(x+1)}{3x+4}$$
$$= \frac{5x-4-x-1}{3x+4}$$
$$= \frac{4x-5}{3x+4}$$

43. $$\frac{4}{x-2}+\frac{x}{2-x} = \frac{4}{x-2}-\frac{x}{x-2} = \frac{4-x}{x-2}$$

44. $$\frac{6}{x-1}-\frac{x}{1-x} = \frac{6}{x-1}+\frac{x}{x-1} = \frac{x+6}{x-1}$$

45. $$\frac{4}{x-1}-\frac{2}{x+2} = \frac{4(x+2)}{(x-1)(x+2)}-\frac{2(x-1)}{(x+2)(x-1)}$$
$$= \frac{4x+8-2x+2}{(x+2)(x-1)}$$
$$= \frac{2x+10}{(x+2)(x-1)}$$
$$= \frac{2(x+5)}{(x+2)(x-1)}$$

46. $$\frac{2}{x+5}-\frac{5}{x-5} = \frac{2(x-5)}{(x+5)(x-5)}-\frac{5(x+5)}{(x+5)(x-5)}$$
$$= \frac{2x-10-5x-25}{(x+5)(x-5)}$$
$$= \frac{-3x-35}{(x+5)(x-5)}$$
$$= -\frac{3x+35}{(x+5)(x-5)}$$

47. $$\frac{x}{x+1}+\frac{2x-3}{x-1} = \frac{x(x-1)}{(x+1)(x-1)}+\frac{(2x-3)(x+1)}{(x-1)(x+1)}$$
$$= \frac{x^2-x+2x^2-x-3}{(x-1)(x+1)}$$
$$= \frac{3x^2-2x-3}{(x-1)(x+1)}$$

48. $$\frac{3x}{x-4}+\frac{2x}{x+3} = \frac{3x(x+3)}{(x-4)(x+3)}+\frac{2x(x-4)}{(x-4)(x+3)}$$
$$= \frac{3x^2+9x+2x^2-8x}{(x-4)(x+3)}$$
$$= \frac{5x^2+x}{(x-4)(x+3)}$$
$$= \frac{x(5x+1)}{(x-4)(x+3)}$$

49. $\dfrac{x-3}{x+2}-\dfrac{x+4}{x-2}=\dfrac{(x-3)(x-2)}{(x+2)(x-2)}-\dfrac{(x+4)(x+2)}{(x-2)(x+2)}$

$=\dfrac{x^2-5x+6-(x^2+6x+8)}{(x+2)(x-2)}$

$=\dfrac{x^2-5x+6-x^2-6x-8}{(x+2)(x-2)}$

$=\dfrac{-11x-2}{(x+2)(x-2)}$ or $\dfrac{-(11x+2)}{(x+2)(x-2)}$

50. $\dfrac{2x-3}{x-1}-\dfrac{2x+1}{x+1}=\dfrac{(2x-3)(x+1)}{(x-1)(x+1)}-\dfrac{(2x+1)(x-1)}{(x+1)(x-1)}$

$=\dfrac{2x^2-x-3-(2x^2-x-1)}{(x+1)(x-1)}$

$=\dfrac{2x^2-x-3-2x^2+x+1}{(x+1)(x-1)}$

$=\dfrac{-2}{(x+1)(x-1)}$

$=-\dfrac{2}{(x+1)(x-1)}$

51. $\dfrac{x}{x^2-4}+\dfrac{1}{x}=\dfrac{x^2+x^2-4}{x\left(x^2-4\right)}$

$=\dfrac{2x^2-4}{x\left(x^2-4\right)}$

$=\dfrac{2\left(x^2-2\right)}{x(x-2)(x+2)}$

52. $\dfrac{x-1}{x^3}+\dfrac{x}{x^2+1}=\dfrac{(x-1)\left(x^2+1\right)+x^4}{x^3\left(x^2+1\right)}$

$=\dfrac{x^3-x^2+x-1+x^4}{x^3\left(x^2+1\right)}$

$=\dfrac{x^4+x^3-x^2+x-1}{x^3\left(x^2+1\right)}$

53. $x^2-4=(x+2)(x-2)$

$x^2-x-2=(x+1)(x-2)$

Therefore, LCM $=(x+2)(x-2)(x+1)$.

54. $x^2-x-12=(x+3)(x-4)$

$x^2-8x+16=(x-4)(x-4)$

Therefore, LCM $=(x+3)(x-4)^2$.

55. $x^3-x=x\left(x^2-1\right)=x(x+1)(x-1)$

$x^2-x=x(x-1)$

Therefore, LCM $=x(x+1)(x-1)$.

56. $3x^2-27=3\left(x^2-9\right)=3(x+3)(x-3)$

$2x^2-x-15=(2x+5)(x-3)$

Therefore, LCM $=3(2x+5)(x-3)(x+3)$.

57. $4x^3-4x^2+x=x\left(4x^2-4x+1\right)$

$=x(2x-1)(2x-1)$

$2x^3-x^2=x^2(2x-1)$

x^3

Therefore, LCM $=x^3(2x-1)^2$.

58. $x-3$

$x^2+3x=x(x+3)$

$x^3-9x=x\left(x^2-9\right)=x(x+3)(x-3)$

Therefore, LCM $=x(x+3)(x-3)$.

59. $x^3-x=x\left(x^2-1\right)=x(x+1)(x-1)$

$x^3-2x^2+x=x\left(x^2-2x+1\right)=x(x-1)^2$

$x^3-1=(x-1)\left(x^2+x+1\right)$

Therefore, LCM $=x(x+1)(x-1)^2\left(x^2+x+1\right)$.

60. $x^2+4x+4=(x+2)^2$

$x^3+2x^2=x^2(x+2)$

$(x+2)^3$

Therefore, LCM $=x^2(x+2)^3$.

61. $\dfrac{x}{x^2-7x+6}-\dfrac{x}{x^2-2x-24}$

$=\dfrac{x}{(x-6)(x-1)}-\dfrac{x}{(x-6)(x+4)}$

$=\dfrac{x(x+4)}{(x-6)(x-1)(x+4)}-\dfrac{x(x-1)}{(x-6)(x+4)(x-1)}$

$=\dfrac{x^2+4x-x^2+x}{(x-6)(x+4)(x-1)}=\dfrac{5x}{(x-6)(x+4)(x-1)}$

62. $\dfrac{x}{x-3}-\dfrac{x+1}{x^2+5x-24}$

$=\dfrac{x}{(x-3)}-\dfrac{x+1}{(x-3)(x+8)}$

$=\dfrac{x(x+8)}{(x-3)(x+8)}-\dfrac{x+1}{(x-3)(x+8)}$

$=\dfrac{x^2+8x-x-1}{(x-3)(x+8)}=\dfrac{x^2+7x-1}{(x-3)(x+8)}$

63. $\dfrac{4x}{x^2-4}-\dfrac{2}{x^2+x-6}$

$=\dfrac{4x}{(x-2)(x+2)}-\dfrac{2}{(x+3)(x-2)}$

$=\dfrac{4x(x+3)}{(x-2)(x+2)(x+3)}-\dfrac{2(x+2)}{(x+3)(x-2)(x+2)}$

$=\dfrac{4x^2+12x-2x-4}{(x-2)(x+2)(x+3)}$

$=\dfrac{4x^2+10x-4}{(x-2)(x+2)(x+3)}$

$=\dfrac{2(2x^2+5x-2)}{(x-2)(x+2)(x+3)}$

64. $\dfrac{3x}{x-1}-\dfrac{x-4}{x^2-2x+1}=\dfrac{3x}{(x-1)}-\dfrac{x-4}{(x-1)^2}$

$=\dfrac{3x(x-1)}{(x-1)(x-1)}-\dfrac{x-4}{(x-1)^2}$

$=\dfrac{3x^2-3x-x+4}{(x-1)^2}$

$=\dfrac{3x^2-4x+4}{(x-1)^2}$

65. $\dfrac{3}{(x-1)^2(x+1)}+\dfrac{2}{(x-1)(x+1)^2}$

$=\dfrac{3(x+1)+2(x-1)}{(x-1)^2(x+1)^2}$

$=\dfrac{3x+3+2x-2}{(x-1)^2(x+1)^2}$

$=\dfrac{5x+1}{(x-1)^2(x+1)^2}$

66. $\dfrac{2}{(x+2)^2(x-1)}-\dfrac{6}{(x+2)(x-1)^2}$

$=\dfrac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2}$

$=\dfrac{2x-2-6x-12}{(x+2)^2(x-1)^2}$

$=\dfrac{-4x-14}{(x+2)^2(x-1)^2}$

$=\dfrac{-2(2x+7)}{(x+2)^2(x-1)^2}$

67. $\dfrac{x+4}{x^2-x-2}-\dfrac{2x+3}{x^2+2x-8}$

$=\dfrac{x+4}{(x-2)(x+1)}-\dfrac{2x+3}{(x+4)(x-2)}$

$=\dfrac{(x+4)(x+4)}{(x-2)(x+1)(x+4)}-\dfrac{(2x+3)(x+1)}{(x+4)(x-2)(x+1)}$

$=\dfrac{x^2+8x+16-(2x^2+5x+3)}{(x-2)(x+1)(x+4)}$

$=\dfrac{-x^2+3x+13}{(x-2)(x+1)(x+4)}$

68. $\dfrac{2x-3}{x^2+8x+7}-\dfrac{x-2}{(x+1)^2}$

$=\dfrac{2x-3}{(x+1)(x+7)}-\dfrac{x-2}{(x+1)^2}$

$=\dfrac{(2x-3)(x+1)}{(x+1)(x+7)(x+1)}-\dfrac{(x-2)(x+7)}{(x+1)^2(x+7)}$

$=\dfrac{2x^2-x-3-(x^2+5x-14)}{(x+1)^2(x+7)}$

$=\dfrac{x^2-6x+11}{(x+1)^2(x+7)}$

69. $\dfrac{1}{x}-\dfrac{2}{x^2+x}+\dfrac{3}{x^3-x^2}$

$=\dfrac{1}{x}-\dfrac{2}{x(x+1)}+\dfrac{3}{x^2(x-1)}$

$=\dfrac{x(x+1)(x-1)-2x(x-1)+3(x+1)}{x^2(x+1)(x-1)}$

$=\dfrac{x(x^2-1)-2x^2+2x+3x+3}{x^2(x+1)(x-1)}$

$=\dfrac{x^3-x-2x^2+5x+3}{x^2(x+1)(x-1)}$

$=\dfrac{x^3-2x^2+4x+3}{x^2(x+1)(x-1)}$

70. $\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^3-x^2}$

$=\dfrac{x}{(x-1)^2}+\dfrac{2}{x}-\dfrac{x+1}{x^2(x-1)}$

$=\dfrac{x^3+2x(x-1)^2-(x+1)(x-1)}{x^2(x-1)^2}$

$=\dfrac{x^3+2x(x^2-2x+1)-(x^2-1)}{x^2(x-1)^2}$

$=\dfrac{x^3+2x^3-4x^2+2x-x^2+1}{x^2(x-1)^2}$

$=\dfrac{3x^3-5x^2+2x+1}{x^2(x-1)^2}$

71. $\dfrac{1}{h}\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)=\dfrac{1}{h}\left(\dfrac{1\cdot x}{(x+h)x}-\dfrac{1(x+h)}{x(x+h)}\right)$

$=\dfrac{1}{h}\left(\dfrac{x-x-h}{x(x+h)}\right)$

$=\dfrac{-h}{hx(x+h)}$

$=\dfrac{-1}{x(x+h)}$

72. $\dfrac{1}{h}\left(\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right)$

$=\dfrac{1}{h}\left(\dfrac{1\cdot x^2}{(x+h)^2x^2}-\dfrac{1(x+h)^2}{x^2(x+h)^2}\right)$

$=\dfrac{1}{h}\left(\dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}\right)$

$=\dfrac{-2xh-h^2}{hx^2(x+h)^2}$

$=\dfrac{h(-2x-h)}{hx^2(x+h)^2}$

$=\dfrac{-2x-h}{x^2(x+h)^2}$

$=-\dfrac{2x+h}{x^2(x+h)^2}$

73. $\dfrac{1+\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{\left(\dfrac{x}{x}+\dfrac{1}{x}\right)}{\left(\dfrac{x}{x}-\dfrac{1}{x}\right)}=\dfrac{\left(\dfrac{x+1}{x}\right)}{\left(\dfrac{x-1}{x}\right)}=\dfrac{x+1}{x}\cdot\dfrac{x}{x-1}=\dfrac{x+1}{x-1}$

74. $\dfrac{4+\dfrac{1}{x^2}}{3-\dfrac{1}{x^2}}=\dfrac{\left(\dfrac{4x^2}{x^2}+\dfrac{1}{x^2}\right)}{\left(\dfrac{3x^2}{x^2}-\dfrac{1}{x^2}\right)}=\dfrac{\left(\dfrac{4x^2+1}{x^2}\right)}{\left(\dfrac{3x^2-1}{x^2}\right)}$

$=\dfrac{4x^2+1}{x^2}\cdot\dfrac{x^2}{3x^2-1}$

$=\dfrac{4x^2+1}{3x^2-1}$

75. $\dfrac{2-\dfrac{x+1}{x}}{3+\dfrac{x-1}{x+1}}=\dfrac{\dfrac{2x}{x}-\dfrac{x+1}{x}}{\dfrac{3(x+1)}{x+1}+\dfrac{x-1}{x+1}}=\dfrac{\dfrac{2x-x-1}{x}}{\dfrac{3x+3+x-1}{x+1}}$

$=\dfrac{\dfrac{x-1}{x}}{\dfrac{4x+2}{x+1}}=\dfrac{x-1}{x}\cdot\dfrac{x+1}{2(2x+1)}$

$=\dfrac{(x-1)(x+1)}{2x(2x+1)}$

76. $$\frac{1-\dfrac{x}{x+1}}{2-\dfrac{x-1}{x}}=\frac{\left(\dfrac{x+1}{x+1}-\dfrac{x}{x+1}\right)}{\left(\dfrac{2x}{x}-\dfrac{x-1}{x}\right)}=\frac{\left(\dfrac{1}{x+1}\right)}{\left(\dfrac{x+1}{x}\right)}$$
$$=\frac{1}{x+1}\cdot\frac{x}{x+1}$$
$$=\frac{x}{(x+1)^2}$$

77. $$\frac{\dfrac{x+4}{x-2}-\dfrac{x-3}{x+1}}{x+1}$$
$$=\frac{\left(\dfrac{(x+4)(x+1)}{(x-2)(x+1)}-\dfrac{(x-3)(x-2)}{(x+1)(x-2)}\right)}{x+1}$$
$$=\frac{\left(\dfrac{x^2+5x+4-(x^2-5x+6)}{(x-2)(x+1)}\right)}{x+1}$$
$$=\frac{10x-2}{(x-2)(x+1)}\cdot\frac{1}{x+1}$$
$$=\frac{2(5x-1)}{(x-2)(x+1)^2}$$

78. $$\frac{\dfrac{x-2}{x+1}-\dfrac{x}{x-2}}{x+3}$$
$$=\frac{\left(\dfrac{(x-2)(x-2)}{(x+1)(x-2)}-\dfrac{x(x+1)}{(x-2)(x+1)}\right)}{x+3}$$
$$=\frac{\left(\dfrac{x^2-4x+4-(x^2+x)}{(x-2)(x+1)}\right)}{x+3}$$
$$=\frac{-5x+4}{(x-2)(x+1)}\cdot\frac{1}{x+3}$$
$$=\frac{-5x+4}{(x-2)(x+1)(x+3)}$$
$$=\frac{-(5x-4)}{(x-2)(x+1)(x+3)}$$

79. $$\frac{\dfrac{x-2}{x+2}+\dfrac{x-1}{x+1}}{\dfrac{x}{x+1}-\dfrac{2x-3}{x}}$$
$$=\frac{\left(\dfrac{(x-2)(x+1)}{(x+2)(x+1)}+\dfrac{(x-1)(x+2)}{(x+1)(x+2)}\right)}{\left(\dfrac{x^2}{(x+1)(x)}-\dfrac{(2x-3)(x+1)}{x(x+1)}\right)}$$
$$=\frac{\left(\dfrac{x^2-x-2+x^2+x-2}{(x+2)(x+1)}\right)}{\left(\dfrac{x^2-(2x^2-x-3)}{x(x+1)}\right)}$$
$$=\frac{\left(\dfrac{2x^2-4}{(x+2)(x+1)}\right)}{\left(\dfrac{-x^2+x+3}{x(x+1)}\right)}$$
$$=\frac{2(x^2-2)}{(x+2)(x+1)}\cdot\frac{x(x+1)}{-(x^2-x-3)}$$
$$=\frac{2x(x^2-2)}{-(x+2)(x^2-x-3)}$$
$$=\frac{-2x(x^2-2)}{(x+2)(x^2-x-3)}$$

80.
$$\frac{\frac{2x+5}{x}-\frac{x}{x-3}}{\frac{x^2}{x-3}-\frac{(x+1)^2}{x+3}}$$
$$=\frac{\left(\frac{(2x+5)(x-3)}{x(x-3)}-\frac{x(x)}{x(x-3)}\right)}{\left(\frac{x^2(x+3)}{(x-3)(x+3)}-\frac{(x-3)(x+1)^2}{(x-3)(x+3)}\right)}$$
$$=\frac{\left(\frac{2x^2-x-15-x^2}{x(x-3)}\right)}{\left(\frac{x^3+3x^2-(x^3-x^2-5x-3)}{(x-3)(x+3)}\right)}$$
$$=\frac{\left(\frac{x^2-x-15}{x(x-3)}\right)}{\left(\frac{4x^2+5x+3}{(x-3)(x+3)}\right)}$$
$$=\frac{x^2-x-15}{x(x-3)}\cdot\frac{(x-3)(x+3)}{4x^2+5x+3}$$
$$=\frac{(x^2-x-15)(x+3)}{x(4x^2+5x+3)}$$

81.
$$1-\frac{1}{1-\frac{1}{x}}=1-\frac{1}{\frac{x-1}{x}}$$
$$=1-\frac{x}{x-1}$$
$$=\frac{x-1-x}{x-1}$$
$$=\frac{-1}{x-1}$$

82.
$$1-\frac{1}{1-\frac{1}{1-x}}=1-\frac{1}{\frac{1-x-1}{1-x}}=1-\frac{1}{\frac{-x}{1-x}}$$
$$=1-\frac{1-x}{-x}=1+\frac{1-x}{x}$$
$$=\frac{x+1-x}{x}$$
$$=\frac{1}{x}$$

83.
$$\frac{2(x-1)^{-1}+3}{3(x-1)^{-1}+2}=\frac{\frac{2}{x-1}+3}{\frac{3}{x-1}+2}=\frac{\frac{2}{x-1}+\frac{3(x-1)}{x-1}}{\frac{3}{x-1}+\frac{2(x-1)}{x-1}}$$
$$=\frac{\frac{2+3(x-1)}{x-1}}{\frac{3+2(x-1)}{x-1}}$$
$$=\frac{2+3(x-1)}{x-1}\cdot\frac{x-1}{3+2(x-1)}$$
$$=\frac{2+3(x-1)}{3+2(x-1)}=\frac{2+3x-3}{3+2x-2}$$
$$=\frac{3x-1}{2x+1}$$

84.
$$\frac{4(x+2)^{-1}-3}{3(x+2)^{-1}-1}=\frac{\frac{4}{x+2}-3}{\frac{3}{x+2}-1}=\frac{\frac{4}{x+2}-\frac{3(x+2)}{x+2}}{\frac{3}{x+2}-\frac{1(x+2)}{x+2}}$$
$$=\frac{\frac{4-3(x+2)}{x+2}}{\frac{3-(x+2)}{x+2}}$$
$$=\frac{4-3(x+2)}{x+2}\cdot\frac{x+2}{3-(x+2)}$$
$$=\frac{4-3(x+2)}{3-(x+2)}=\frac{4-3x-6}{3-x-2}$$
$$=\frac{-3x-2}{-x+1}=\frac{3x+2}{x-1}$$

85.
$$\frac{(2x+3)\cdot 3-(3x-5)\cdot 2}{(3x-5)^2}=\frac{6x+9-6x+10}{(3x-5)^2}$$
$$=\frac{19}{(3x-5)^2}$$

86.
$$\frac{(4x+1)\cdot 5-(5x-2)\cdot 4}{(5x-2)^2}=\frac{20x+5-20x+8}{(5x-2)^2}$$
$$=\frac{13}{(5x-2)^2}$$

87. $$\frac{x\cdot 2x-\left(x^2+1\right)\cdot 1}{\left(x^2+1\right)^2}=\frac{2x^2-x^2-1}{\left(x^2+1\right)^2}$$
$$=\frac{x^2-1}{\left(x^2+1\right)^2}$$
$$=\frac{(x-1)(x+1)}{\left(x^2+1\right)^2}$$

88. $$\frac{x\cdot 2x-\left(x^2-4\right)\cdot 1}{\left(x^2-4\right)^2}=\frac{2x^2-x^2+4}{\left(x^2-4\right)^2}=\frac{x^2+4}{\left(x^2-4\right)^2}$$
$$=\frac{x^2+4}{(x+2)^2(x-2)^2}$$

89. $$\frac{(3x+1)\cdot 2x-x^2\cdot 3}{(3x+1)^2}=\frac{6x^2+2x-3x^2}{(3x+1)^2}$$
$$=\frac{3x^2+2x}{(3x+1)^2}$$
$$=\frac{x(3x+2)}{(3x+1)^2}$$

90. $$\frac{(2x-5)\cdot 3x^2-x^3\cdot 2}{(2x-5)^2}=\frac{6x^3-15x^2-2x^3}{(2x-5)^2}$$
$$=\frac{4x^3-15x^2}{(2x-5)^2}$$
$$=\frac{x^2(4x-15)}{(2x-5)^2}$$

91. $$\frac{\left(x^2+1\right)\cdot 3-(3x+4)\cdot 2x}{\left(x^2+1\right)^2}=\frac{3x^2+3-6x^2-8x}{\left(x^2+1\right)^2}$$
$$=\frac{-3x^2-8x+3}{\left(x^2+1\right)^2}$$
$$=\frac{-\left(3x^2+8x-3\right)}{\left(x^2+1\right)^2}$$
$$=-\frac{(3x-1)(x+3)}{\left(x^2+1\right)^2}$$

92. $$\frac{\left(x^2+9\right)\cdot 2-(2x-5)\cdot 2x}{\left(x^2+9\right)^2}=\frac{2x^2+18-4x^2+10x}{\left(x^2+9\right)^2}$$
$$=\frac{-2x^2+10x+18}{\left(x^2+9\right)^2}$$
$$=\frac{-2\left(x^2-5x-9\right)}{\left(x^2+9\right)^2}$$

93. $$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$\frac{1}{f}=(n-1)\left(\frac{R_2+R_1}{R_1\cdot R_2}\right)$$
$$\frac{R_1\cdot R_2}{f}=(n-1)\left(R_2+R_1\right)$$
$$\frac{f}{R_1\cdot R_2}=\frac{1}{(n-1)\left(R_2+R_1\right)}$$
$$f=\frac{R_1\cdot R_2}{(n-1)\left(R_2+R_1\right)}$$
$$f=\frac{0.1(0.2)}{(1.5-1)(0.2+0.1)}$$
$$=\frac{0.02}{0.5(0.3)}=\frac{0.02}{0.15}=\frac{2}{15}\text{ meters}$$

94. $$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{R_2R_3+R_1R_3+R_1R_2}{R_1R_2R_3}$$
$$R=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$
$$=\frac{5\cdot 4\cdot 10}{4\cdot 10+5\cdot 10+5\cdot 4}$$
$$=\frac{200}{110}=\frac{20}{11}\text{ ohms}$$

95. $1+\dfrac{1}{x}=\dfrac{x+1}{x}\Rightarrow a=1,b=1,c=0$

$$1+\frac{1}{1+\frac{1}{x}}=1+\frac{1}{\left(\frac{x+1}{x}\right)}=1+\frac{x}{x+1}$$
$$=\frac{x+1+x}{x+1}=\frac{2x+1}{x+1}$$
$$\Rightarrow a=2,b=1,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}=1+\frac{1}{\left(\frac{2x+1}{x+1}\right)}=1+\frac{x+1}{2x+1}$$
$$=\frac{2x+1+x+1}{2x+1}=\frac{3x+2}{2x+1}$$
$$\Rightarrow a=3,b=2,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}=1+\frac{1}{\left(\frac{3x+2}{2x+1}\right)}=1+\frac{2x+1}{3x+2}$$
$$=\frac{3x+2+2x+1}{3x+2}=\frac{5x+3}{3x+2}$$
$$\Rightarrow a=5,b=3,c=2$$

If we continue this process, the values of a, b and c produce the following sequences:

$a: 1,2,3,5,8,13,21,....$

$b: 1,1,2,3,5,8,13,21,.....$

$c: 0,1,1,2,3,5,8,13,21,.....$

In each case we have a *Fibonacci Sequence*, where the next value in the list is obtained from the sum of the previous 2 values in the list.

96. Answers will vary.

97. Answers will vary.

Section R.8

1. 9; –9

2. 4; $|-4|=4$

3. index

4. True

5. cube root

6. False; $\sqrt[4]{(-3)^4}=|-3|=3$

7. $\sqrt[3]{27}=\sqrt[3]{3^3}=3$

8. $\sqrt[4]{16}=\sqrt[4]{2^4}=2$

9. $\sqrt[3]{-8}=\sqrt[3]{(-2)^3}=-2$

10. $\sqrt[3]{-1}=\sqrt[3]{(-1)^3}=-1$

11. $\sqrt{8}=\sqrt{4\cdot 2}=2\sqrt{2}$

12. $\sqrt[3]{54}=\sqrt[3]{27\cdot 2}=3\sqrt[3]{2}$

13. $\sqrt[3]{-8x^4}=\sqrt[3]{-8x^3\cdot x}=-2x\sqrt[3]{x}$

14. $\sqrt[4]{48x^5}=\sqrt[4]{16x^4\cdot 3x}=2x\sqrt[4]{3x}$

15. $\sqrt[4]{x^{12}y^8}=\sqrt[4]{\left(x^3\right)^4\left(y^2\right)^4}=x^3y^2$

16. $\sqrt[5]{x^{10}y^5}=\sqrt[5]{\left(x^2\right)^5y^5}=x^2y$

17. $\sqrt[4]{\dfrac{x^9y^7}{xy^3}}=\sqrt[4]{x^8y^4}=x^2y$

18. $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}}=\sqrt[3]{\dfrac{1}{27x^3}}=\dfrac{\sqrt[3]{1}}{\sqrt[3]{27x^3}}=\dfrac{1}{3x}$

19. $\sqrt{36x}=6\sqrt{x}$

20. $\sqrt{9x^5}=3\sqrt{x^4\cdot x}=3x^2\sqrt{x}$

21. $\sqrt{3x^2}\sqrt{12x}=\sqrt{36x^2\cdot x}=6x\sqrt{x}$

22. $\sqrt{5x}\sqrt{20x^3}=\sqrt{100x^4}=10x^2$

23. $\left(\sqrt{5}\sqrt[3]{9}\right)^2=\left(\sqrt{5}\right)^2\left(\sqrt[3]{9}\right)^2$
$$=5\cdot\sqrt[3]{9^2}=5\sqrt[3]{81}=5\cdot 3\sqrt[3]{3}=15\sqrt[3]{3}$$

24. $\left(\sqrt[3]{3}\sqrt{10}\right)^4=\left(\sqrt[3]{3}\right)^4\left(\sqrt{10}\right)^4$
$$=\sqrt[3]{3^4}\cdot 10^2=3\sqrt[3]{3}\cdot 100=300\sqrt[3]{3}$$